Modeling of underground structures in rocksalt using a unified viscoplastic model

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The mechanical behavior of rocksalt has been investigated extensively over the past four decades or so. Many constitutive models have been formulated to describe some of its main features, such as time and history dependency. One of these has been proposed by the authors and collaborators about 10 years ago, and since then it has been refined to better describe important aspects of rocksalt inelastic response to complex loading paths. This model, called SUVIC, is based on the unified theory of inelastic flow where a single set of equations (and material parameters) is used for different loading conditions. It makes use of internal state variables (ISV) to define hardening and recovery of the material microstructural state during inelastic flow. In this paper, it is shown how the latest version of SUVIC can describe fairly complex mechanical responses. Then, using a 2D/3D object oriented finite element code in which the SUVIC equations have been implemented, the authors illustrate its application for actual underground structures.

1. INTRODUCTION

Modeling of the inelastic behavior of salt is a critical issue in the design of hyperstatic structures such as underground openings in rocksalt deposits. Until fairly recently, such analysis were frequently carried out using steady-state creep laws assuming inelastic flow without memory effects. The most often used models to assess the creep (time dependent) behavior of rocksalt are based on a power law equation of the following type:

$$\dot{\varepsilon}_s = A_1\ \sigma^n f(T) \qquad (1)$$

where $\dot{\varepsilon}_s$ is the steady-state deviatoric strain rate, σ is the deviatoric stress, A_1 and n are experimentally determined parameters, and $f(T)$ is a temperature function which is usually assumed to be the Arrhenius law. With this equation however, the values of material parameters are not constant and usually vary with the loading state [1-2]. Another limitation of this formulation is that it does not take into account the transient nature of inelastic behavior of rocksalt, or its strain-hardening. This phenomenon, which is a true material property, must be distinguished from the transient deformation behavior of a structure which is caused, in part, by the stress redistribution due to non linear material response during inelastic flow [3].

A widely used approach to describe both the transient and steady-state flow of polycrystalline materials, including rocksalt, is based on a partitioned formulation defining the creep (inelastic) strain rate as:

$$\dot{\varepsilon}^i = \dot{\varepsilon}_t + \dot{\varepsilon}_s \qquad (2a)$$

where $\dot{\varepsilon}_t$, the transient component, is often given by a strain-hardening expression of the following type :

$$\dot{\varepsilon}_t = A_2\ \sigma^n (\varepsilon_i)^b \qquad (2b)$$

A_2 and b also represent material constants. Sometimes a third component is added to Equation (2a) for tertiary creep, a phenomenon related to microcracking and material damage. This aspect will not be considered in this paper which only deals with ductile (fully plastic isovolumetric) flow; information on damage modeling of rocksalt is given elsewhere [4].

The partitioned approach further requires that other terms be added to describe short term straining, usually through an elasto-plastic component with yield criteria, a flow rule and a plastic potential. Unfortunately, with such partitioned approach, the model equations and corresponding parameters can only be valid under the conditions used for their determination (but which may not correspond to the actual in situ conditions). To alleviate this limitation and hopefully generalize the model applicability, various strategies have been elaborated. For instance, it has been proposed that a creep law be combined to a so-called relaxation law, two limiting cases that, in principle, encompass a wide range of loading conditions [5]. The inconsistency and limitations of this and of other similar partitioned expressions have been discussed at length by the authors in previous publications [3,6].

An alternative approach, initially developed for metallic materials, is to rely on a unified formulation and to introduce evolutionary internal state variables (ISV) in the model to represent a broader range of material responses, including transient and steady-state flow.

2. UNIFIED MODELING

Constitutive modeling of rocksalt behavior requires a proper representation of the viscous nature and of memory effects in the constitutive equations. An interesting approach is to rely on internal state variables associated to specific mechanical characteristics of the material. For each ISV, an evolution law can be developed to describe the competitive action of hardening and recovery processes. The ISV model presented below is an updated version of SUVIC (**S**train rate history dependent **U**nified **V**iscoplastic model with **I**nternal state variables for **C**rystalline materials), a model developed for rocksalt behavior in the context of rate-dependent plasticity [7]. It uses three types of ISV to describe mixed (kinematic and isotropic) hardening : a tensorial back stress (B_{ij}), an isotropic yield strength (R) and a normalizing scalar drag stress (K). During transient inelastic flow, the internal state variables evolve and reach saturation before or at steady-state, where all rate variables (internal and external) become constant and codirectional. In the following, a new hyperbolic sine law is used to describe the relationship between $\dot{\varepsilon}^i$ and σ at steady-state [8,9].

Within the unified theory framework, material hardening and the resulting mechanical state are entirely described at a given time by the current value of the external (observable) state variables ($\sigma,\ \dot{\varepsilon},\ T$) and of the appropriately selected internal (hidden) state variables (here B_{ij}, R and K). This approach allows a good representation of inelastic responses associated to different types of behavior including plasticity, creep and relaxation.

2.1. Basic concepts

The proposed constitutive equations, developed within the unified inelasticity theory framework, consider that only elastic and viscoplastic straining occurs. The model presented here, referred to as $SUVIC_{sh}$ (an acronym for SUVIC with an hyperbolic sine law), is an extension of the model proposed initially by Aubertin and coworkers [7].

With this ISV modeling approach, the material structural state is described using the so-called internal stress σ_i. The concept of a macroscopic (mean) internal stress has been proposed decades ago for inelastic models to better describe material response to external loadings [10]. It has been found especially useful when the common power-law exponent of the stationary (steady-state, secondary) creep function n (see Equation (1)) increases substantially with the stress level. This internal stress σ_i generally opposes the action of the applied deviatoric stress σ. Its value can be estimated by appropriate measurement techniques usually involving a sudden stress decrease. When introduced in the constitutive equations, the internal stress appears explicitly in the basic kinetic law, so that one can write for uniaxial monotonous loading [6-8]:

$$\dot{\varepsilon}^i = A\left\langle\frac{\sigma-\sigma_i}{K}\right\rangle^N \tag{3}$$

where $\dot{\varepsilon}^i$ is the inelastic strain rate, and σ_i is the internal stress that opposes the applied stress σ. The normalizing parameter K has already been defined as the drag stress, while A and N are material constants; $<\ >$ are the MacAuley brackets ($\langle x\rangle = 1/2\left(x+|x|\right)$).

The existence of a macroscopic internal stress acting within a representative volume element of crystalline materials has been rationalized using the heterogeneous nature of dislocations distribution, which produces 'hard' and 'soft' regions with different local stresses and different dislocations mobility [11].

The model equations presented below include three distinct internal state variables with updated evolution laws. Two variables, R and K (the scalar yield strength and the drag stress respectively), account for isotropic effects, while B (the tensorial back stress) accounts for kinematic (or flow induced anisotropic) effects. The combination of B and R constitute the internal stress σ_i. Each of the three ISV evolves according to phenomenological expressions with competing components for strain hardening, strain induced dynamic recovery, and time/thermally induced static recovery.

2.2. Set of equations

For small perturbations under isothermal loading, the total strain rate tensor $\dot{\varepsilon}_{ij}$ is the sum of elastic and inelastic strain rate tensors:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{e} + \dot{\varepsilon}_{ij}^{i} \tag{4}$$

The elastic strain rate tensor is given by the generalized Hooke's law for isotropic materials. The inelastic component of the strain rate tensor is given by a single kinetic law obtained from a multiaxial generalization of equation (3):

$$\dot{\varepsilon}_{ij}^{i} = A\left\langle \frac{X_{ae} - R}{K} \right\rangle^{N} n_{ij} \tag{5a}$$

with

$$X_{ae} = \left\| S_{ij} - B_{ij} \right\| = \left[\frac{3}{2} \left(S_{ij} - B_{ij} \right) \left(S_{ij} - B_{ij} \right) \right]^{\frac{1}{2}} \tag{5b}$$

$$n_{ij} = \frac{3}{2} \frac{S_{ij} - B_{ij}}{\left\| S_{ij} - B_{ij} \right\|} \tag{5c}$$

$$S_{ij} = \sigma_{ij} - \delta_{ij}\sigma_m \tag{5d}$$

where σ_{ij} is the stress tensor; S_{ij} is the deviatoric stress tensor; σ_m is the mean stress, δ_{ij} is the Kronecker delta; n_{ij} is a unit directional tensor that gives the inelastic strain rate the same orientation as the active stress tensor. In these equations, the kinematic internal variable B is the sum of separately evolving constituents, B_s (for short range effect) and B_l (for long range effect):

$$B_{ij} = \sum_{\alpha = s,l} B_{ij,\alpha} = B_{ij,s} + B_{ij,l} \tag{6}$$

Two such components are commonly used to represent the kinematic backstress in unified models. It is considered here that the short range back stress, B_s, grows much faster and saturates much sooner than its long range counterpart, B_l.

Phenomenologically, each ISV is assumed to evolve through a competitive process of the following type, given for a generic internal state variable Y ($\in B_{ij,s}$, $B_{ij,l}$, R, K):

$$\dot{Y} = f_h - f_d - f_s \tag{7}$$

where f_h, f_d and f_s are respectively the hardening term that accounts for strengthening (hardening) mechanisms, and the dynamic and static recovery terms that account for softening (recovery) mechanisms. The hardening and dynamic recovery terms both evolve with inelastic strain. The static recovery term, on the other hand, evolves with time and cumulated value of Y. The functional forms giving the evolution of each particular term depend on the type of internal state variable. They are given with full details in references [8,9].

Within the adopted modeling framework, it is considered that steady-state flow is independent of previous mechanical events, or memory effects. Therefore, it can be formulated using only external variables. Because steady-state flow is associated, at the microscopic level, to dislocations climb, glide and/or cross slip mechanisms, the relationship between $\dot{\varepsilon}_e$ and σ_e' can be represented by an hyperbolic sine function, so that one can write:

$$\dot{\varepsilon}_{e}^{i} = \dot{\varepsilon}_{o} \sinh^{n}\left(\frac{\sigma_e'}{\sigma_o} \right) \tag{8}$$

where σ_o, $\dot{\varepsilon}_o$ and n are material constants for steady-state

In the preceding equations, the well-known von Mises equivalencies are used:

$$\sigma_e = \left[\frac{3}{2} S_{ij} S_{ij} \right]^{\frac{1}{2}} \tag{9}$$

$$\dot{\varepsilon}_{e}^{i} = \left[\frac{2}{3} \dot{\varepsilon}_{ij}^{i} \dot{\varepsilon}_{ij}^{i} \right]^{\frac{1}{2}} \tag{10}$$

The good correlation between equation (8) and measured values obtained on Avery Island salt submitted to creep and CSR tests at different temperatures has been confirmed by results presented by Julien [9].

3. APPLICATION TO VARIOUS TESTING CONDITIONS

To illustrate some of the $SUVIC_{sh}$ model capabilities for a variety of loading conditions, the above equations are applied to experimental test results on Avery Island (AI) salt. The results include constant strain rate (CSR) tests and creep tests performed at a confining pressure of 15 MPa and published by Senseny et al. [12] with full details. The parameter values were first estimated using a physical approach based on test results that emphasize specific aspects of the material response [6]. The preliminary values are then refined with an identification code based on the least-square method with a combination of several minimization techniques [8,9]. The ensuing parameters values are given in Table 1.

Figure 1 shows stress-strain curves at different strain rates; the ones calculated from the model compare well to experimental results. These CSR test results show that the effect of strain rate on material hardening is very important. This aspect is well reproduced by the model. This demonstrates that short term transient behavior of rocksalt is well represented by the evolution laws of the internal state variables.

In many cases, tests with long durations and low strain rates are of interest, especially in mining and civil engineering applications.

For Avery Island rocksalt, creep tests with rapid changes in the strain rate due to quick increases or decreases in the stress state (Figure 2) have also been performed. As can be seen on this figure, the model equations are equally able to represent this type of response using the same set of material parameters given in Table 1. Other applications for CSR tests and creep tests with complex loading path, and even for relaxation tests are also presented in Julien [9].

Since the same set of material constants is used to reproduce the various experimental results, it can be said that the $SUVIC_{sh}$ model equations adequately reflect how the AI salt respond to these different loading conditions.

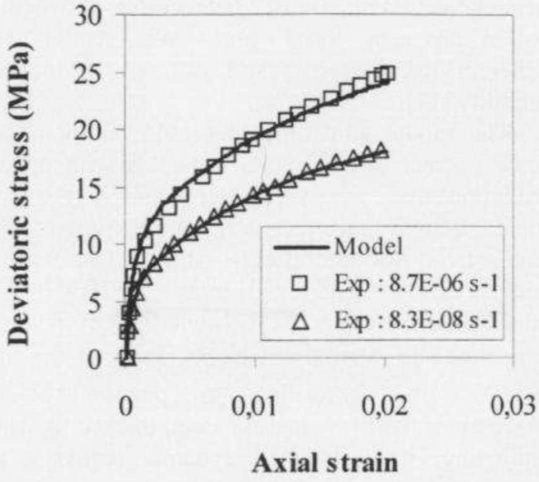

Figure 1. Measured and calculated response of AI rocksalt samples during CSR loading, showing the effect of strain rate; data from Senseny et al. [12].

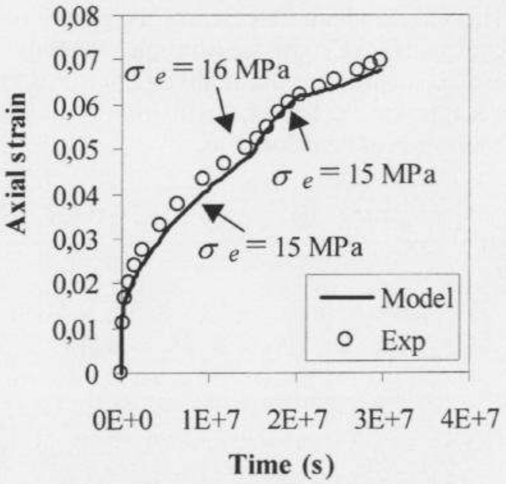

Figure 2. Measured and calculated response of AI rocksalt sample during creep test with varying applied deviatoric stress; data from Senseny et al. [12].

Table 1. Parameter values obtained for Avery Island rocksalt; the full set of equations is given in references [8,9].

Parameters	Values
E	31000 MPa
ν	0.38
A	0.176 10^{-5} s^{-1}
A_{1s}	20395 MPa
A_{1l}	1218 MPa
A_{2s}	0.104 10^{-2} MPa s^{-1}
A_{2l}	0.458 10^{-15} MPa s^{-1}
A_3	95 MPa
A_4	0.456 10^{-8} MPa s^{-1}
A_5	27 MPa
A_6	0.543 10^{-13} MPa s^{-1}
B_{0s}	1.47 MPa
B_{0l}	3.37 MPa
R_0	3.04 MPa
σ_0	9.15 MPa
$\dot{\varepsilon}_O$	0.135 10^{-10} s^{-1}
N	4
n	3
m	1

Initial values of ISV (at $\varepsilon^i = 0$) : B_s=B_l=R=0 ; K=1 MPa.

4. APPLICATION TO UNDERGROUND STRUCTURES

The equations presented above can be used also for calculations on man-made excavations. For that purpose, the finite element program ZéBuLoN was selected for the numerical implementation of the model. This code has been developed at the École des Mines de Paris, France [13,14]. Until recently, it had mostly been used for research purposes on thermo-mechanical problems for metallic materials.

The current version of ZéBuLoN used here, written in C++, has been developed with modular programming to provide better control of data, and a high modularity to allow quick implementation of new algorithms using predefined structures. This version provides a suitable computational framework for geomechanical problems with inclusion of non-linearities arising from material behavior, loading sequences and mesh refinement upon localization if necessary.

For illustration purposes, the evolution of the stress distribution for two parallel galleries at 200 m level in a salt mine has been simulated. The two galleries are considered to be long compared to the square cross section (8mx8m). The pillar between them is 13 m wide. The overburden pressure is about 4.5 MPa. The calculation time period was limited to 2 millions seconds (23 days). The gallery on the left-side has smooth corners while the second gallery has sharp ones. Modeling of the excavation was carried out on a vertical cross section normal to the axis of the galleries. The whole structure comprises 1707 nodal points with 524 8-node elements. The model parameters identified for Avery Island rocksalt were used for the calculations.

Figure 3 shows the distribution of Mises equivalent stress at the end. It can be seen that the asymmetry of the results arises from the difference in smoothness of the corners.

The results shown here just illustrate some of the constitutive and numerical capabilities of the model. Other structural calculations are presented in Julien [9] and in related (unpublished) reports.

5. CONCLUSION

A modeling framework, based on the unified theory of inelastic flow using internal state variables (ISV), is presented. The $SUVIC_{sh}$ model, developed in such context, is then introduced. The model is applied to different laboratory test results on Avery Island salt. A single set of parameters is able to describe different types of test results, including CSR and creep tests. It is shown that the inelastic behavior of rocksalt, like that of other crystalline materials, exhibits a unique (unified) inelastic behavior that can be expressed by a single kinematic law with accompanying evolution laws for the ISV. The authors have implemented $SUVIC_{sh}$ model into a 2D/3D finite element code, which can now be applied for structural calculations pertaining to underground structures. A sample calculation is shown.

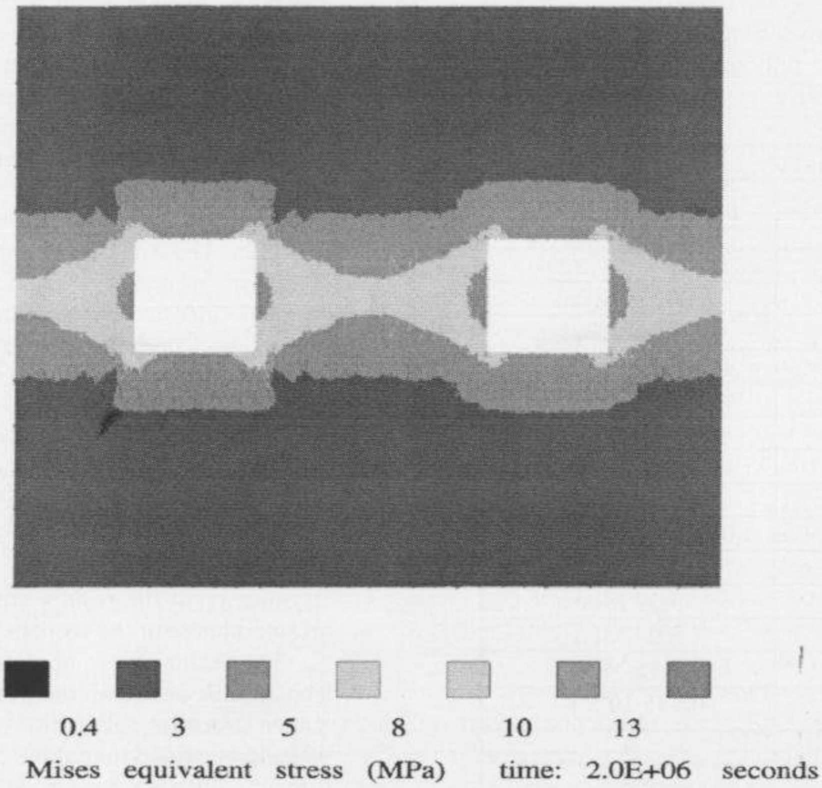

Figure 3. The stress distribution around two parallel galleries at 200-m level in a salt (the gallery on the left-side has rounded corners and the second one on the right-side has sharp edges).

REFERENCES

1. W.R. Wawersik and D.H. Zeuch, Tectonophysics, 121 (1986), 125-152.
2. U. Hunsche and A. Hampel, Engineering Geology, 52 (1999), 271-291.
3. M. Aubertin, D.E. Gill and B. Ladanyi, Proc. 7th Symp. on Salt, Kyoto, I, (1993), 93-104.
4. M. Aubertin, J. Sgaoula, S. Servant , M. Julien, D.E. Gill and B. Ladanyi, Proc. 4th Conf. on the Mechanical Behavior of Salt,Trans Tech. Pub., Clausthal-Zellerfeld (1998), 205-220.
5. M. Haupt, Rock Mechanics and Rock Engineering 24 (1991), 179-206.
6. M. Aubertin, M.R. Julien, S. Servant and D.E. Gill. Canadian Geotechnical Journal, 36 (1999), no 3 (to be published).
7. M. Aubertin, D.E. Gill and B. Ladanyi, Mechanics of Materials 11 (1991), 63-82.
8. M. Aubertin, O.M.L. Yahya and M. Julien, Int. J. of Plasticity, 15 (1999) 1067-1088.
9. M.R. Julien, Une modélisation constitutive et numérique du comportement mécanique du sel gemme. Ph.D. Thesis, Ecole Polytechnique de Montréal (1999).
10. E. Orowan, The Creep of Metals. J. Wet. Scot. Iron Soc. Inst. 54 (1947), 45-68.
11. H. Mughrabi, Acta Met. 31 (1983), 1367-1379.
12. P.E. Senseny, N.S. Brodsky, and K.L. De Vries, J. of Engng Mat. Techno., ASME, 115 (1993), 157-162.
13. R. Foerch, J. Besson, G. Cailletaud and P. Pilvin, Comput. Methods Appl. Mech. Eng., 141 (1997), 355-372.
14. J. Besson, and R. Foerch, Comput. Methods Appl. Mech. Eng., 142 (1997), 165-18.